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Complex Mass in Nonrelativistic Quantum Mechanics

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The use of complex mass in nonrelativistic quantum mechanics and its relation to the influence of vacuum field fluctuation in electron motion are discussed.

1. INTRODUCTION

In Pardy's work (Pardy, 1973) an attempt was made to generalize Feynman's known integral by the introduction of complex mass. The author in his work defines this integral as an infinite iteration product of functions of the type $K(x_{k+1}, t_{k+1}; x_k, t_k)$ multiplied by functions of type $W(x_{k+1}, t_{k+1}; x_k, t_k)$ (Morette, 1951). The probability amplitude $U(x_A, t_A; x_B, t_B)$ is then given

$$U(x_A, t_A; x_B, t_B) = \lim_{n\to\infty} \int_{-\infty}^{\infty} \cdots \int \prod_{k=0}^{n-1} K(x_{k+1}, t_{k+1}; x_k, t_k) \times W(x_{k+1}, t_{k+1}; x_k, t_k) \prod_{k=0}^{n} dx_k \delta(x_0 - x_A)\delta(x_n - x_B) \tag{1.1}$$

x_{k+1}, x_k in equation (1.1) are two one-dimensional points corresponding to an interval $t_{k+1} - t_k = \varepsilon$ infinitesimally small. With regard to factors K and

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W the author defines them as follows:

$$K(x_{k+1}, t_{k+1}; x_k, t_k) = \frac{1}{A} \exp\left\{ \frac{i}{\hbar} \left[\frac{m}{2} \left(\frac{x_{k+1} - x_k}{\varepsilon} \right)^2 - V(x_{k+1}) \right] \varepsilon \right\} \tag{1.2}$$

$$W(x_{k+1}, t_{k+1}; x_k, t_k) = \frac{1}{(4\pi D\varepsilon)^{1/2}} \exp\left[-\frac{(x_{k+1} - x_k)^2}{4D\varepsilon} \right] \tag{1.3}$$

Expression (1.2) describes the motion of a quantum particle (provided with mass m) in the Coulomb potential $V(x_{k+1})$ from point x_k to x_{k+1} in infinitesimal time interval ε, whereas expression (1.3) describes the probability that the classical particle characterized by a diffusion constant D arrives aided by the Brownian motion in the time interval ε from x_k to x_{k+1}.

The key point of Pardy's work is his statement that the influence of vacuum field fluctuation (electromagnetic field, scalar field, etc.) on the quantum particle motion in the Coulomb potential $V(x_{k+1})$ from x_k to x_{k+1} in the infinitesimal time interval ε can be expressed by

$$U(x_{k+1}, t_{k+1}; x_k, t_k) = K \cdot W \tag{1.4}$$

Because the quantal motion of a particle, as described by the equation (1.2), is corrected here by the classical motion of the same particle as defined by equation (1.3) which is supposed, according to the author, to include the quantum motion of vacuum field fluctuation, expression (1.4) is without any physical content.

2. DISCUSSION

Now it is a trivial matter to show that (i) the assumption (1.4) is identical to the introduction of the complex mass concept as discussed in Nelson's well-known work since 1964 (Nelson, 1964); and (ii) the complex mass in the case considered, of the motion of an electron in the Coulomb field, does not include the influence of vacuum fluctuation.

To prove point (i) it is enough to write the equation (1.4) as follows:

$$U(x_{k+1}, x_k; \varepsilon) = \frac{1}{A\sqrt{(4\pi D\varepsilon)^{1/2}}} \exp\left\{ \frac{i}{\hbar} \left[\frac{M}{2} \left(\frac{x_{k+1} - x_k}{\varepsilon} \right)^2 - V(x_{k+1}) \right] \varepsilon \right\} \tag{2.1}$$

where

$$M = m + \frac{i\hbar}{2D} \tag{2.2}$$

The normalization constant A in (2.1) can be determined from unitary condition (Morette, 1951)

$$\int_{-\infty}^{\infty} U^*(x_{k+1}, x_k; \varepsilon) U(x_{k+1}, x_k'; \varepsilon)\, dx_{k+1} = \delta(x_k - x_k') \tag{2.3}$$

and then the equation (1.1) can be written symbolically

$$U(x_A, t_A; x_B, t_B) = \int \exp\left\{ \frac{i}{\hbar} S[x(t)] \right\} \mathfrak{D}[x(t)] \tag{2.4}$$

where

$$S[x(t)] = \frac{1}{2} M \int_{t_A}^{t_B} \dot{x}^2(t)\, dt - \int_{t_A}^{t_B} V[x(t), t]\, dt \tag{2.5}$$

is the Riemannian sum of infinitesimal actions

$$S[x_{k+1}, x_k; \varepsilon] = \left[\frac{1}{2} M \left(\frac{x_{k+1} - x_k}{\varepsilon} \right)^2 - V(x_{k+1}) \right] \varepsilon \tag{2.6}$$

and owing to the fulfillment of condition (2.3) the equation (2.4) has mathematical meaning.

Nelson, starting from (2.2), proved the following. (Nelson, 1964; see also Morette, 1969).

(1) For M purely imaginary, i.e., $D = i\hbar/2M > 0$, the equation (2.4) is equivalent to the heat equation with a purely imaginary potential $iV(x)$:

$$\frac{\partial \psi}{\partial t} = D \frac{\partial^2 \psi}{\partial x^2} - iV(x)\psi \tag{2.7}$$

and its solution is given by Wiener's integral. In this case equation (2.4) is reduced to Wiener's integral with purely imaginary potential $iV(x)$.

(2) For M real the integral (2.4) exists for almost every real value of the mass parameter M (except for M, which creates a set of Lebesgue measure 0).

(3) For M complex the integral (2.4) exists for all complex values of the parameter M.

Point (3) assured the existence of the integral (2.4) in case of complex M. It is now a trivial matter to show that equation (2.4) does not include the influence of vacuum fluctuation on the electron motion.

Let us rewrite the equation (2.4) to form

$$U(x_A, t_A; x_B, t_B) = \int_{x_A}^{x_B} \exp\left\{ \frac{i}{\hbar} \int_{t_A}^{t_B} \left[\frac{1}{2} m\dot{x}^2(t) - V(x(t), t) \right] dt + \frac{i}{\hbar} I_P \right\} \mathfrak{D}x(t) \tag{2.8}$$

with

$$I_P = \frac{i\hbar}{4D} \int_{t_A}^{t_B} \dot{x}^2(t)\, dt \tag{2.9}$$

and compare with the corresponding expression which includes the vacuum fluctuation and leads to the correct value of Lamb's shift in the case of the electromagnetic field (Feynman, Hibbs, 1965; Morette, 1969) or in case of the scalar fluctuation field (Nelson, 1963):

$$I_{\text{electromag}} = \frac{1}{2} \sum_{\vec{\mathbf{k}}} \int \frac{4\pi}{2k_c} \left[j_{1\mathbf{k}}(t) j^*_{1\mathbf{k}}(s) + j_{2\mathbf{k}}(t) j^*_{2\mathbf{k}}(s) \right] e^{-ik_c|t-s|}\, dt\, ds \tag{2.10}$$

$$I(k)_{\text{scal}} = \frac{ig^2}{4\omega(k)} \int \cos k \cdot [x(t) - x(s)] e^{-i\omega(k)|t-s|}\, dt\, ds \tag{2.11}$$

Relations (2.10) and (2.11) are valid for the interaction of a single particle with fluctuating electromagnetic or scalar field. $j_{1\mathbf{k}}$ and $j_{2\mathbf{k}}$ are components of current density of the considered particle which are perpendicular to wave vector $\mathbf{k}$, and $kc = \omega$ is the frequency of the considered fluctuating vacuum field. If constant D in relation (2.9) has the meaning of diffusion constant characterizing the Brownian motion of a classical particle, then comparing expression (2.9) with (2.10) we come to the conclusion that I_P does not contain an interaction of the considered particle with a fluctuating vacuum field. This is physically obvious especially from the fact that in equation (2.9) Feynman's famous propagator of the particle in an electromagnetic field is missing, i.e.,

$$e^{-ikc|t-s|} = 2ikc \int_{-\infty}^{\infty} \frac{e^{i\omega|t-s|}}{\omega^2 - k^2c^2 + i\varepsilon} \frac{d\omega}{2\pi} \tag{2.12}$$

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